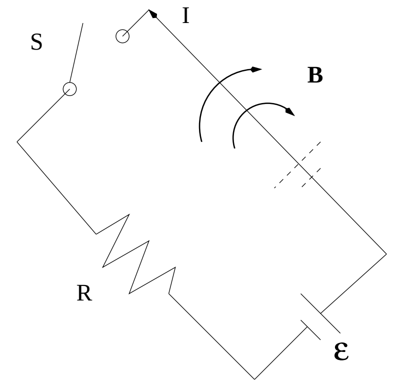
## 1 – Self-inductance

- Consider a circuit consisting of a switch, a resistor, and an emf.




- When the switch is closed, the current does not change immediately from zero to its maximum value $\varepsilon/R$ but only increases gradually.
- As the current increases with time, so does the magnetic flux through the loop (which is due to the current in the loop).



Inductance and Alternating Current Circuits

- Lenz's law $\Rightarrow$ the induced emf in the loop is opposite to the direction of the current. The opposing emf results in only a gradual increase of the current.
- This effect, i.e. that a changing current induces an emf in the same circuit, is called **self-inductance**.
- For the same reason, if the switch is opened, the current only gradually decreases to zero.
- Faraday's law: $\varepsilon = -N\,\frac{\Delta\Phi}{\Delta t}$
  The magnetic flux is proportional to the magnetic field, which is proportional to the current in the circuit.

# Inductance and Alternating Current Circuits

- Thus, the self-induced emf is proportional to the time rate of change of the current:

$$\boxed{\varepsilon = -L\,\frac{\Delta I}{\Delta t}} \qquad (1)$$

$L$ is called the **inductance** of the device. The SI unit of inductance is the **henry (H)**: $1\ H = 1\ V \cdot s/A$

- Relation between self-inductance and magnetic flux: $N\,\frac{\Delta\Phi}{\Delta t} = L\,\frac{\Delta I}{\Delta t}$

$$\boxed{L = \frac{N\Phi}{I}} \qquad (2)$$

- The emf induced by an inductor prevents a battery from establishing a current in a circuit instantaneously. The battery has to do work to produce a current. The energy stored by an inductor is

$$\boxed{\text{Energy} = \frac{1}{2}\,LI^2} \qquad (3)$$



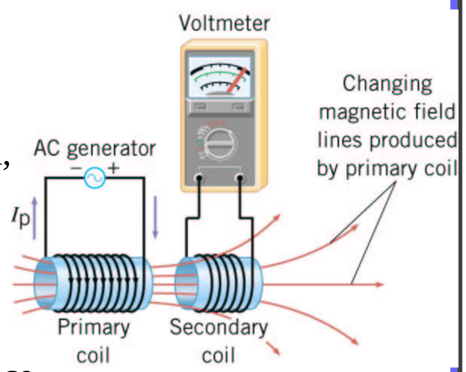
# Inductance and Alternating Current Circuits

## 2 – Mutual Inductance

The effect that a changing current in one circuit (=primary circuit) induces an emf in another circuit (=secondary circuit) is called **mutual inductance**.

Faraday's law: $\varepsilon_s = -N_s\,\frac{\Delta\Phi_s}{\Delta t}$
The magnetic flux in the secondary circuit, $\Phi_s$, is proportional to the alternating magnetic field, which is proportional to the alternating current in the primary circuit:

$$\boxed{N_s\Phi_s = MI_p.}$$

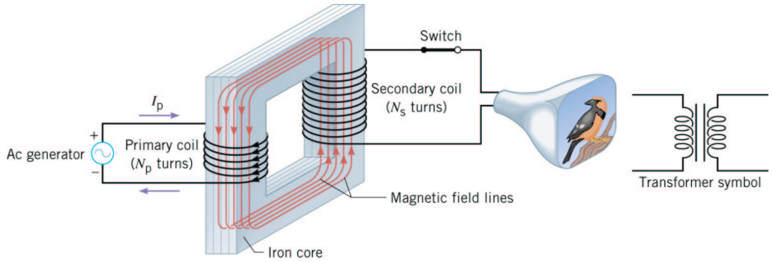



$M$ is called the **mutual inductance** of the device.

Thus, the emf due to mutual inductance is proportional to the time rate of change of the primary current:

$$\boxed{\varepsilon_s = -M\,\frac{\Delta I_p}{\Delta t}} \qquad (4)$$

## 3 – Transformers




Transformers are used to increase or decrease an AC voltage through self-inductance and mutual inductance: Alternating current (AC voltage) $\Rightarrow$ changing magnetic flux in both coils, $\Delta\Phi/\Delta t$, $\Rightarrow$ emf is induced in the primary coil due to self inductance and in the secondary coil due to mutual inductance: $\varepsilon_s = -N_s \Delta\Phi/\Delta t$ and $\varepsilon_p = -N_p \Delta\Phi/\Delta t$. Assuming the resistances in the coils are negligible, we find

$$\boxed{\frac{\varepsilon_s}{\varepsilon_p} = \frac{V_s}{V_p} = \frac{N_s}{N_p}} \qquad (5)$$

$V_{s,p}$: Voltage across (secondary,primary) coil.





## 4 – Inductors in an AC Circuit

An inductor

- is a device that operates on the basis of Faraday's law of electromagnetic induction,
- it develops a voltage that opposes a change in the current.

Consider a circuit consisting of an inductor and an AC generator:

- The changing current output of the generator produces an induced emf in the inductor of magnitude

$$V = -L\,\frac{\Delta I}{\Delta t}$$

- The effective resistance of an inductor in an AC circuit is measured by the **inductive reactance**, $X_L$, defined by

$$\boxed{X_L = 2\pi f L} \qquad (6)$$

- The analog of Ohm's law for the average voltage across an inductor is:

$$\boxed{V_{rms} = I_{rms} X_L} \quad (7)$$

- When a sinusoidal voltage is applied across an inductor, $V = V_0 \sin(2\pi ft)$, the voltage reaches its maximum value, $V_0$, one quarter of a cycle before the current reaches its maximum.

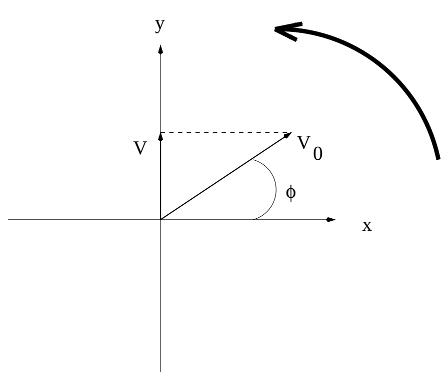
**The current lags behind the voltage by $90°$**





**Phasors and Phasors Diagrams:**

The voltage across each element in a circuit can be represented by a rotating vector.

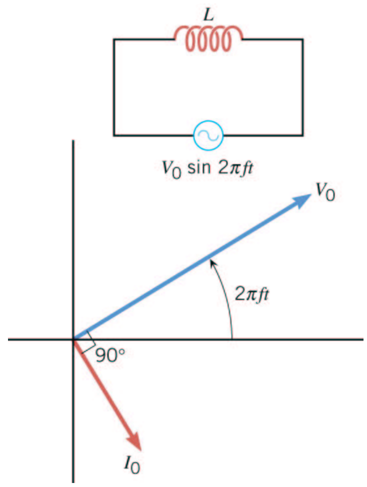



The rotating vectors are called **phasors**; the diagram is called a **phasor diagram**. The diagram represents the voltage given by the expression

$$V = V_0 \sin(2\pi ft + \phi)$$

with $\phi$ being the phase angle between the voltage and the current.

Phasor diagram for an inductor in an AC circuit:





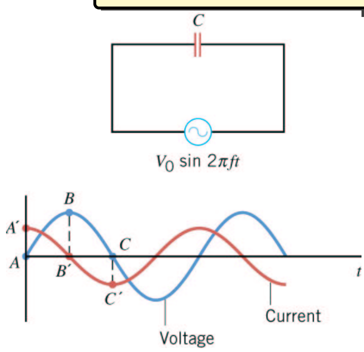

Inductance and Alternating Current Circuits

## 5 – Capacitors in an AC Circuit

Consider a circuit consisting of a capacitor and an AC generator

- The voltage reaches its maximum value **one quarter of a cycle after** the current reaches its maximum. It is common to say:

**The current leads the voltage by** 90°



- The impeding effect of a capacitor on the current in an AC circuit is expressed in terms of a factor called the **capacitive reactance**, $X_C$, defined as

$$\boxed{X_C = \frac{1}{2\pi f C}} \tag{8}$$

- A DC source can be considered an AC source with $f = 0$. For $f = 0$, $X_C = \infty$ and the current is zero.

- Analog form of Ohm's law for a capacitor in an AC **(!)** circuit:

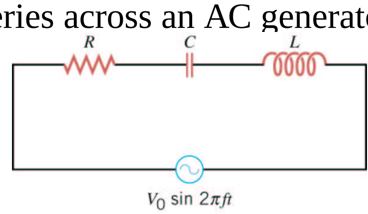
$$\boxed{V_{rms} = I_{rms} X_C} \tag{9}$$





## 6 – The RLC Series Circuit

Now consider a circuit containing a resistor, an inductor, and a capacitor connected in series across an AC generator:

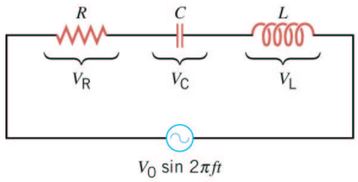



The (instantaneous) current varies with time according to $I = I_0 \sin(2\pi f t)$. The instantaneous voltages across the three elements have the following phase relations to the instantaneous current:

1. The instantaneous voltage across the resistor, $V_R$, is *in phase* with the instantaneous current.
2. The instantaneous voltage across the inductor, $V_L$, *leads* the current by 90°.
3. The instantaneous voltage across the capacitor, $V_C$, *lags behind* the current by 90°.

The net instantaneous voltage, $V$, across all three elements is the sum of the three individual instantaneous voltages: $V = V_R + V_C + V_L$.

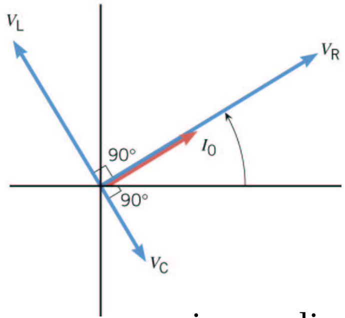
The total peak (or average) voltage can be found using the phasor diagram as:




$V_0 = \sqrt{V_R^2 + (V_L - V_C)^2}$

The phase angle is given by:

$\tan\Phi = \frac{V_L - V_C}{V_R}$




**Note:** All voltages are peak voltages. But the same expression applies to average voltages, since $V_{rms} = V_0/\sqrt{2}$, $I_{rms} = I_0/\sqrt{2}$.



# Inductance and Alternating Current Circuits

In the following all voltages are average voltages:

Inserting $V_R = I_{rms}R$, $V_L = I_{rms}X_L$ and $V_C = I_{rms}X_C$ we find:

$$V_{rms} = \sqrt{V_R^2 + (V_L - V_C)^2} = I_{rms}\sqrt{R^2 + (X_L - X_C)^2}$$

Defining the **impedance**, $Z$ by

$$\boxed{Z = \sqrt{R^2 + (X_L - X_C)^2}} \qquad (10)$$

we find the analog of Ohm's law for an RLC circuit

$$\boxed{V_{rms} = I_{rms}Z} \qquad (11)$$

## 7 – Power in an AC Circuit

The only element in an RLC circuit which dissipates energy is the resistor. The average power lost in a resistor is

$$\overline{P} = I_{rms}^2 R = I_{rms} V_R$$

$V_R$ can be expressed in terms of the voltage of the source and the phase angle:

$$V_R = V_{rms} \cos \Phi$$

Hence the average power dissipated in an AC circuit is

$$\boxed{\overline{P} = I_{rms} V_{rms} \cos \Phi} \qquad (12)$$



Inductance and Alternating Current Circuits

## 8 – Resonance in a Series RLC Circuit

The average current in a series RLC circuit can be written as

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{V_{rms}}{\sqrt{R^2 + (X_L - X_C)^2}}$$

The current has its maximum when $Z$ is at a minimum. This happens for $X_C = X_L$. If this condition is satisfied, $Z = R$. The frequency $f = f_0$ at which it happens is called the **resonance frequency** of the circuit. From $X_C = X_L$ at $f = f_0$ one finds

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}$$

Solve for the resonance frequency $f_0$:

$$\boxed{f_0 = \frac{1}{2\pi\sqrt{LC}}} \qquad (13)$$